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COMPARISON BETWEEN LP AND GP MODELS ON SMALL SCALE INDUSTRY

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1. Abstract

The Small Scale Industry (SCI) plays a very important role in planning and solving decision problems. The mathematical model designed to help business manager's plan and make the necessary decision to allocated resources. The mathematical model is structured into two sub-models, the first based on Linear Programming model and the second based on Priority Weighted Goal Programming model. The aim of this paper is to present the comparison differences of Linear Programming (LP) and Priority Weighted Goal Programming (PWGP) to optimize Goal constraints of a farm. The results illustrate that PWGP more suitable model as of a customer satisfaction perspective than LP model.

Key words: Linear Programming, Goal Programming, SCI.

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2. Introduction

Small Scale Industrial sector has emerged as a domestic device of expansion in several developing and developed economies of the world. They have emerged as a vibrant and dynamic sector of country economy by virtue of their significant contribution to GDP and industrial production management developments. Several mathematical programming models have been applied to assist the production management planning problems. In this criterion Linear Programming and Goal Programming methods are deity multi criteria decision models. Chambers and Charnes (1961) pioneered the development of a deterministic Linear Programming model in assets and liability. Akpan, N. P et al.(2016), Application of Linear Programming for Optimal Use of Raw Materials in Bakery. This work utilized the concept of simplex algorithm; an aspect of Linear Programming to allocate raw materials to competing variables in bakery for the purpose of profit maximization.

However, sometimes, Bushra Abdul Halim (2015), the decision makers stated, multiple criteria in their managerial problems, the Linear Programming model is unable to combine all the criteria simultaneously. Therefore, the Goal Programming technique has been introduced in order to solve multi-objective problems. Ignizio (1976) proposed a Goal programming model to analyze multiple conflicting objectives while taking into account the constraints and preference of the decision maker.

Standard GP models, Cinzia Colapinto (2015), deal with deterministic Goals that are precisely defined. Variants to standard GP models includes lexicographic GP (LGP) where the model is optimized according to DM's prioritized choice, and weighted GP (WGP) where positive and negative deviations from Goals can differ according to the importance of the objectives.

3. Review of the Literature

Today, using and applying mathematical techniques in Small Scale Industry (SCI) plays a very important role in planning and solving decision problems. One of the most commonly used mathematical techniques is Linear Programming (LP), which was first used by Waugh (1951) in optimizing feed rations. LP is frequently used in cases when one target function is defined and usually minimizes the costs and maximizes the profit. According to Steven J Miller (2007), Linear Programming is a generalization of Linear algebra use in modelling so many real life problems ranging from scheduling airline routes to shipping oil from refineries to cities for the purpose of finding inexpensive diet capable of meeting daily requirements. Miller argued that the reason for the great versatility of Linear Programming is due to the ease at which constraints can be incorporated into the Linear Programming model. Also used LP, Balogun et al. (2012), in production sectors is the problem of management, that many companies are faced with decision relating to the use of limited resources such as manpower, raw materials, capital etc.

The major limitation of the LP mathematical approach is optimize one goal at a time, because of this disadvantage LP is not suitable to use in the SCI planning process having more than one goal must be optimized. To avoid this shortcoming, Jernej (2014) some developed models were upgraded by another mathematical approach called weighted Goal Programming (WGP), where the numerous objective functions (goals) were optimized. One of the specific properties of the WGP techniques is using weights for creating the hierarchical tree of the preferred goals and penalty functions to keep the goals inside tolerant bounds. The advantages of linking LP and WGP are that LP is used to minimize or maximize the goals separately, and WGP reaches all of the Goals from the LP sub-models at the same time. A positive feature of the GP philosophy is its simplicity and ease of use, Aouni B. (2001), which justifies its wide popularity for solving multi criteria decision making models in diverse fields. Many More adopted the Goal Programming models for maximize the production planning problems; Leung and Chan (2009) propose a GP model for aggregate production planning with resource utilization constraint. In this paper, adopted a Priority Weighted Goal Programming with percentage normalisation model to improve the goal values according to the product satisfaction.

4. Model formulation

The general **Linear Programming** model [4] with n decision variables and m constraints can be stated in the following form

$$optimize\ (\max\ \text{ or } \min)\ Z=\sum_{j=1}^{n} c_j x_j \ ....(objective\ function)$$

$s.t$

$$\sum_{j=1}^{n} a_{ij} x_j (\le, =, \ge) b_i,\ i=1,2,3,\ldots,m$$

$$x_j \ge 0,\ j=1,2,3,\ldots,n.$$

Where c_j represent the per unit profit (or cost) of decision variables x_j to the value of the objective function. And a_{ij} represent the amount of resource consumed per unit of the decision variables and b_i represents the total availability of the ith resource. Z represents the measure of performance which can be either profit, or cost or reverence etc.

The Linear Programming Problems are single objective oriented problems and the constrained sets of LPP are hard constraints which never accept the violation. Only one single objective is dealt with while in real life situations, problems come with multi-objectives. Under Linear Programming to increase production by a single process the quantity of all inputs is to be increased in a fixed proportion. But the production of a number of goods can be increased to some extent by increasing only one or two inputs. It means that production can be increased to some extent by varying factors proportion. In spite of these limitations, consider the model formulation of Goal Programming.

The general **Goal Programming** model [5] considered as follows.

(1) $\min z=\sum_{i=1}^{m}(d_i^+ + d_i^-);$ i=1, 2, 3, ..., m

st

(2) $\sum_{j=1}^{n} c_{i,j} y_j - d_i^+ + d_i^- = A_i;$ j=1, 2, 3, ..., n

(3) $\sum_{j=1}^{n} c_{i,j} y_j \begin{vmatrix} \ge \\ = \\ \le \end{vmatrix} A_i;$ i=m+1, ..., m+p

(4) $y_j, d_i^+, d_i^- \ge 0$

It is important to note that:

Equation (1) is referred to objective function, which is the summation of all deviational variables. Equations (2) and (3) are called Goal and system constraint functions; and they are both referred to Linear constrain function and Equation (4) is non-negativity constraint.

- m is the number of goals,
- p = number of structural constraints
- n = number of decision variables.
- z = the objective function expressed as the summation of all the deviational variables.
- y_j = the jth decision variables.
- $c_{i,j}$ = the coefficient of the *jth* decision variables in the *ith* goal.
- d_i^+, d_i^- are amount of deviation below and above aspiration level respectively.

Also called underachievement and overachievement variables. Therefore, in typical GP model, there are two variables: decision and deviational.

- A_i is the aspiration level

In this paper, it is suggested that priority and weighted Goal Programme with percentage normalisation is given as GP model. It can be stated mathematically in the following form

$$\min Z=\sum_{i=1}^{m}\left(\frac{p_i w_{i,}^- d_i^-}{k_i}+\frac{p_i w_i^+ d_i^+}{k_i}\right) w_i^+, w_{i,}^- \geq 0$$

Subject to the constraint functions of equations (2), (3), and non-negativity restriction of (4) and k_i is the normalization factor.

Therefore, the procedures for achieving a goal are either Minimize the underachievement or Minimize the overachievement or both.

5. Methods and Materials

In this paper, consider the comparison of Linear Programming problem and Goal Programming problem. The data for this research project was collected Gorretta bakery limited, Nigeria, Akpan et al. (2016), given bellow table 1. The data consist of total amount of raw materials (sugar, flour, yeast, salt, and wheat gluten and soybean oil) available for daily production of three different sizes of bread (big loaf, giant loaf and small loaf) and profit contribution per each unit size of bread produced

Raw material	Products			
	Big loaf	Gaint loaf	small loaf	
Profit (N)	30	40	20	20385
Flour (kg)	0.2	0.24	0.14	200
Sugar (g)	0.14	0.2	0.16	160
Yeast (kg)	0.02	0.02	0.02	20
salt(g)	0.0011	0.00105	0.00017	8.5
Wheat Gluten (g)	0.000167	0.002	0.00012	15
soyabean oil (L)	0.015	0.021	0.0098	10

Table 1: Ingredients and profit per unit product data

5.1 LP Model formulation

Let the quantity of big loaf to be produce = x1

Let the quantity of giant loaf to be produce = x2

Let the quantity of small loaf to be produce = x3

Let Z denote the profit to be maximize

The Linear Programming model for the above production data is given by

$$\max\ Z=30x_1+40x_2+20x_3 \le 20385$$

Subject to

$$0.2x_1+0.24x_2+0.14x_3 \le 200$$
$$0.14x_1+0.2x_2+0.16x_3 \le 160$$
$$0.02x_1+0.02x_2+0.02x_3 \le 20$$
$$0.0011x_1+0.00105x_2+0.00017x_3 \le 8.5$$
$$0.000167x_1+0.002x_2+0.00012x_3 \le 15$$
$$0.015x_1+0.0214x_2+0.0098x_3 \le 10$$

$$x_1, x_{2,} x_3 \ge 0$$

5.2 Result and Analysis of Linear Programming Model

The Linear Programming problem solved by Lingo software. The result is show in table 2, the objective function value is 28,385 N, and x_1 =38.0, x_2 =0, x_3 =962 based on this data the optimal solution is derived from the model indicate that two products should be produced 38 units of big loaf and 962 units of small loafs. Their production quantities should be N 20,385. It seems that the dissatisfaction of a customer choose at least a giant loaf, for the reason that the customer satisfaction also impacts on regular profits. Overcoming this disadvantage now, consider the Priority Weighted Goal Programming model

LINEAR PROGRAMING PROBLEM BY LINGO

Global optimal solution found.

Objective value:	20384.62
Infeasibilities:	0.000000
Total solver iterations:	3
Elapsed runtime seconds:	0.46
Model Class:	LP
Total variables:	4
NonLinear variables:	0
Integer variables:	0
Total constraints:	7
NonLinear constraints:	0
Total nonzeros:	21
NonLinear nonzeros:	0

Variable	Value	Cost
X1	38.46154	0.000000
X2	0.000000	365.0000
X3	961.5385	0.000000
X	0.000000	0.000000

Table 2: Solution of LPP

5.3 GP Model formulation

Here, consider the achievable profit for these products are N 28,385 and also consider the tolerance limit for these three products should be x_1=250, x_2 = 200 and x_3=550. Then the pre-emptive priority weighted Goal Programming is modelled as

$$\min Z = P_2\left(\frac{1}{20385}n_1\right) + P_2\left(\frac{1}{250}n_2 + \frac{1}{200}n_2 + \frac{1}{550}n_2\right)$$

Soft goals are subject to

$$30x_1 + 40x_2 + 20x_3 + n_1 - p_1 = 20385$$
$$x_1 + n_2 - p_2 = 250$$
$$x_2 + n_3 - p_3 = 200$$
$$x_3 + n_4 - p_4 = 550$$

Hard constrained goals are subject to

$$0.2x_1 + 0.24x_2 + 0.14x_3 \le 200$$
$$0.14x_1 + 0.2x_2 + 0.16x_3 \le 160$$
$$0.02x_1 + 0.02x_2 + 0.02x_3 \le 20$$
$$0.0011x_1 + 0.00105x_2 + 0.00017x_3 \le 8.5$$
$$0.000167x_1 + 0.002x_2 + 0.00012x_3 \le 15$$
$$0.015x_1 + 0.0214x_2 + 0.0098x_3 \le 10$$

$x_i, n_j, p_j \ge 0$ i=1,2,3. j=1,2,3,4.

The priority factors; in the Goal Programming algorithm that follows it is assumed that the priority ranking is absolute i.e., P1 goals are more important than P2 goals and P2 goal will not be achieved until P1 goal have been achieved; same is true for P3, P4 and P5 goals according to their weights and normalization factor are necessary in order to scale all the goals onto the same units of measurement.

5.4 Result and Analysis of Goal Programming Model

The priority weighted Goal programming (PWGP) problem solved by Lingo Software. The result is shown in table 3, also x_1=250.0, x_2=4.0, x_3=550.0 based on this data, the Gorretta bakery limited, choose 250 small loafs, 4 giant loafs and 550 big loafs by that optimize their profit, and the objective function value is z= 1.063958,it means that the priorities are not achieved, at least one of the goals is not met with an underachievement value n2=196, produce 4 giant loafs and n1=1721, means that the profit is N 18664. But a customer can choose at least a giant loaf.

Global optimal solution found.

Objective value:	1.063958
Infeasibilities:	0.000000
Total solver iterations:	1
Elapsed runtime seconds:	0.08
Model Class:	LP
Total variables:	11
NonLinear variables:	0
Integer variables:	0
Total constraints:	11
NonLinear constraints:	0
Total nonzeros:	36
NonLinear nonzeros:	0

Variable	Value	Cost
W1	1.000000	0.000000
W2	1.000000	0.000000
W3	1.000000	0.000000
W4	1.000000	0.000000
X(1)	250.0000	0.000000
X(2)	4.095238	0.000000

X(3)	550.0000	0.000000
N(1)	1721.190	0.000000
N(2)	0.000000	0.4974368E-02
N(3)	195.9048	0.000000
N(4)	0.000000	0.2474391E-02
P(1)	0.000000	0.4905568E-04
P(2)	0.000000	0.9743684E-03
P(3)	0.000000	0.5000000E-02
P(4)	0.000000	0.6562096E-03

Table 3. Solution of PWGP

6. Conclusion

The analysis presented here provides the optimal solutions for different variations of the LP and PWGP models, Because of weighted priorities the optimal solution for the PWGP model is different from the solutions suggested by LP model. LP model suggests maximum of profit without product satisfaction but the PWGP model suggests the product satisfaction with good enough profit. The PWGP satisfies all the goals; therefore it is more suitable to the Small Scale Industries rather compare to LP. In general, the results of this paper make a suitable contribution to understand the possibilities of the Linear Programming and Priority Weighted Goal Programming as a feasible solution to optimizing processes on a Small Scale Industry.

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